International Journal of Theoretical Physics, Vol. 3, No. 5 (1970), pp. 413–424

Electric Charge Systems Moving with the Velocity of Light and Associated with Charge Creation

LL. G. CHAMBERS

Mathematics Department, University College of North Wales, Bangor, Wales

Received: 3 *April* 1970

Abstract

A discussion is given of certain electromagnetic fields associated with charges moving with the velocity of light which are associated with zero magnetic field, and the creation of charge. The stress energy tensor associated with charge creation is also discussed and it is shown that the stress energy tensor includes a term which may be interpreted as a shear.

1. *Introduction*

Recently, Bonnor (1969) has discussed the motion of charge systems moving with the speed of light. It is the purpose of this paper to show that, if charge creation is allowed, solutions of Maxwell's modified equations exist which involve the flow of charge with the speed of light. Before proceeding to the analysis, the equations governing an electromagnetic system with charge creation will be recapitulated. These are due to Watson (1945), and involve the introduction into Maxwell's equations of a scalar creation field N. (In the following a factor μ_0 has been introduced so as to give N the same dimensions as $\mathbf{H}$, the magnetic field.) The Georgi system of units and the usual notation are used. Maxwell's equations modify to

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} - \nabla N \tag{1.1a}$$

$$\nabla . \mathbf{D} = \rho + \frac{1}{c^2}\frac{\partial N}{\partial t} \tag{1.1b}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \tag{1.1c}$$

$$\nabla . \mathbf{B} = 0 \tag{1.1d}$$

The energy density of the field is

$$\tfrac{1}{2}(\epsilon_0 E^2 + \mu_0 H^2 + \mu_0 N^2) \tag{1.2a}$$

The Poynting vector giving the energy flux becomes

$$\mathbf{E} \times \mathbf{H} - \mathbf{E}N \tag{1.2b}$$

and the equation of motion of a charged particle with momentum $\mathbf{p}$ is given by

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B} + \mu_0 \mathbf{v} N) \tag{1.2c}$$

$\mathbf{v}$ being the velocity of the particle. A discussion of the energy momentum stress tensor does not seem to have been given previously, and this is given in Appendix 1.

The rate of charge creation per unit volume is

$$\nabla^2 N - \frac{1}{c^2}\frac{\partial^2 N}{\partial t^2}$$

Suppose now that it be assumed that the charge system is moving with velocity $c\hat{\mathbf{u}}$ where $\hat{\mathbf{u}}$ is a unit vector which does not change with time. This will hold for a fixed direction, the direction radially out from a fixed line, or the direction radially out from a fixed point. Then

$$\mathbf{J} = c\rho\hat{\mathbf{u}} \tag{1.3a}$$

and

$$\mathbf{0} = \mathbf{D} + c^{-1}\{\hat{\mathbf{u}} \times \mathbf{H} + \hat{\mathbf{u}}N\} \tag{1.3b}$$

Equation (1.3b) follows from equation (1.2c) and the fact that if a particle is travelling with the speed of light in a fixed direction, its momentum does not alter. For convenience, $\mathbf{E}$ and $\mathbf{B}$ are replaced everywhere by $\mathbf{D}$ and $\mathbf{H}$.

The energy density of the field is

$$\frac{1}{2}\left(\frac{D^2}{\epsilon_0} + \mu_0 H^2 + \mu_0 N^2\right)$$

and using equation (1.3b), this becomes

$$\tfrac{1}{2}\mu_0\{|\hat{\mathbf{u}} \times \mathbf{H}|^2 + H^2 + 2N^2\} \tag{1.4a}$$

The Poynting vector becomes

$$\frac{1}{\epsilon_0 c}\{\mathbf{H} \times [(\hat{\mathbf{u}} \times \mathbf{H}) + \hat{\mathbf{u}}N] + (\hat{\mathbf{u}} \times \mathbf{H} + \hat{\mathbf{u}}N)N\}$$
$$= \sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\{(H^2 + N^2)\hat{\mathbf{u}} - (\mathbf{H}.\hat{\mathbf{u}})\mathbf{H}\} \tag{1.4b}$$

$\mathbf{J}$ and ρ may be eliminated from equations (1.1a), (1.1b) and (1.3a) to give an equation

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \left(\nabla.\mathbf{D} - \frac{1}{c^2}\frac{\partial N}{\partial t}\right)c\hat{\mathbf{u}} - \nabla N \tag{1.5}$$

It will be seen from equations (1.3b) and (1.5) that $\mathbf{D}$ can be expressed as the sum of $\mathbf{D}_1$ (for which N is zero) and $\mathbf{D}_2$ (for which $\mathbf{H}$ is zero). It is fields of the first type which have been discussed previously (Bonnor, 1969; Chambers, 1970) and accordingly only fields of the second type, for which $\mathbf{H}$ is assumed zero, will be discussed here. Composite fields follow by addition.

2. *Longitudinal Electric Fields*

When $\mathbf{H}$ is zero, equation (1.3b) becomes

$$\mathbf{D} = -c^{-1} N\hat{\mathbf{u}} \tag{2.1}$$

and equation (1.1d) is satisfied identically. $\mathbf{D}$ is parallel to $\hat{\mathbf{u}}$, the direction of flow and the field is therefore of longitudinal electric type.

Equation (1.5) becomes

$$\frac{1}{c}\frac{\partial}{\partial t}(N\hat{\mathbf{u}}) + \frac{1}{c}\frac{\partial N}{\partial t}\hat{\mathbf{u}} + \nabla N + [\nabla.(N\hat{\mathbf{u}})]\,\hat{\mathbf{u}} = 0$$

that is

$$\frac{2}{c}\frac{\partial N}{\partial t}\hat{\mathbf{u}} + 2\nabla N + N(\nabla.\hat{\mathbf{u}})\,\hat{\mathbf{u}} = 0 \tag{2.2}$$

$\hat{\mathbf{u}}$ does not vary with time.

Equation (1.1c) becomes

$$\nabla \times \mathbf{E} = 0 \tag{2.3}$$

The current density $\mathbf{J}$ is given by

$$-\frac{1}{c}\frac{\partial N}{\partial t}\hat{\mathbf{u}} - \nabla N = \tfrac{1}{2}N(\nabla.\hat{\mathbf{u}})\,\hat{\mathbf{u}} \tag{2.4}$$

The energy density becomes $\mu_0 N^2$ and the Poynting vector

$$\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\,N^2\,\hat{\mathbf{u}} \tag{2.5}$$

Consider now what happens when $\hat{\mathbf{u}}$ is one of the three unit vectors referred to previously.

2.1. *Flow in Direction of the z Axis*

In this case $\hat{\mathbf{u}} = \mathbf{k}$, unit vector in the z direction. From equation (2.3) it follows that

$$\nabla \times E\mathbf{k} = -\mathbf{k} \times \nabla E = 0 \tag{2.1.1}$$

and so E is independent of x and y and depends on z and t only.

Thus equation (2.2) becomes

$$\frac{2}{c}\frac{\partial N}{\partial t}\mathbf{k} + 2\frac{\partial N}{\partial z}\mathbf{k} = 0 \tag{2.1.2}$$

It follows immediately that

$$N = f\left(t - \frac{z}{c}\right) \tag{2.1.3}$$

where f is an arbitrary function

$$\mathbf{D} = -\frac{1}{c} f\left(t - \frac{z}{c}\right)\mathbf{k} \tag{2.1.4}$$

The energy density is

$$\mu_0\left[f\left(t - \frac{z}{c}\right)\right]^2$$

and the Poynting vector

$$\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\left[f\left(t - \frac{z}{c}\right)\right]^2\mathbf{k}$$

The current density is given by

$$\begin{aligned} \mathbf{J} &= \tfrac{1}{2}N(\nabla \cdot \hat{\mathbf{u}})\,\hat{\mathbf{u}} = \tfrac{1}{2}N(\nabla \cdot \mathbf{k})\,\mathbf{k} \\ &= \mathbf{0} \end{aligned} \tag{2.1.5}$$

and so the charge density is also zero.

The charge density creation rate is

$$\nabla^2 N - \frac{1}{c^2}\frac{\partial^2 N}{\partial t^2} = \frac{\partial^2 N}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 N}{\partial t^2} \tag{2.1.6}$$

Thus the charge density rate is zero, in apparent contraction to the original hypothesis.

However, the creation field defined by equation (2.1.3) may be regarded as associated with charge creation at infinity, in exactly the same way as a uniform electrostatic field may be regarded as caused by charges at infinity. This will be shown in Appendix 2.

2.2. *Flow Radially Out From the z Axis*

In this case $\hat{\mathbf{u}} = \hat{\mathbf{r}}$ and it follows from equation (2.3) that

$$\mathbf{0} = \nabla \times (E\hat{\mathbf{r}}) = \frac{\partial E}{\partial z}\hat{\boldsymbol{\phi}} - \frac{1}{r}\frac{\partial E}{\partial \phi}\mathbf{k} \tag{2.2.1}$$

where cylindrical polar coordinates are used. Thus E is independent of z and ϕ and so depends on r and t only.

Now $\nabla \cdot \hat{\mathbf{r}} = r^{-1}$, and so equation (2.2) becomes

$$\frac{2}{c}\frac{\partial N}{\partial t}\hat{\mathbf{r}} + \frac{N\hat{\mathbf{r}}}{r} = \mathbf{0} \tag{2.2.2}$$

It follows immediately that

$$N = \frac{f[t-(r/c)]}{r^{1/2}} \tag{2.2.3}$$

where f is an arbitrary function.

$$\mathbf{D} = -\frac{1}{cr^{1/2}} f\left(t - \frac{r}{c}\right)\hat{\mathbf{r}} \tag{2.2.4}$$

The energy density is

$$\frac{\mu_0}{r}\left[f\left(t-\frac{r}{c}\right)\right]^2$$

and the Poynting vector

$$\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\frac{1}{r}\left[f\left(t-\frac{r}{c}\right)\right]^2$$

The current density is given by

$$\begin{aligned}\mathbf{J} &= \tfrac{1}{2}N(\nabla\cdot\hat{\mathbf{u}})\hat{\mathbf{u}} = \tfrac{1}{2}N(\nabla\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} \\ &= \frac{1}{2}\frac{N}{r}\hat{\mathbf{r}} = \frac{1}{2}\frac{f[t-(r/c)]}{r^{3/2}}\hat{\mathbf{r}}\end{aligned} \tag{2.2.5}$$

and the charge density by

$$\frac{1}{2cr^{3/2}} f\left(t-\frac{r}{c}\right)$$

The charge density creation rate is given by

$$\begin{aligned}\nabla^2 N - \frac{1}{c^2}\frac{\partial^2 N}{\partial t^2} &= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N}{\partial r}\right) - \frac{1}{c^2}\frac{\partial^2 N}{\partial t^2} \\ &= 0\end{aligned} \tag{2.2.6}$$

Here again the charge density creation rate is apparently zero. However, consider the flow of charge per unit length out of a cylinder of radius a whose axis is the z axis. This is given by

$$2\pi a[J]_{r=a} = \frac{\pi}{a^{1/2}} f\left(t-\frac{a}{c}\right) \tag{2.2.7}$$

and as a tends to zero this becomes infinite.

Thus, there is an infinite creation of charge along the z axis. The energy per unit length within the cylinder of radius a is given by

$$\int_0^a \mu_0\, 2\pi r \frac{1}{r}\left[f\left(t-\frac{r}{c}\right)\right]^2 = 2\pi\mu_0 \int_0^a f\left(t-\frac{r}{c}\right)^2 dr \tag{2.2.8}$$

The net rate of flow of energy per unit length out of the same cylinder is given by

$$2\pi a\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\frac{1}{a}\left[f\left(t-\frac{a}{c}\right)\right]^2=2\pi\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}[f(t)]^2 \tag{2.2.9}$$

when a tends to zero.

2.3. *Flow Radially Out From Origin*

In this case $\hat{\mathbf{u}}=\hat{\mathbf{r}}$ and it follows from equation (2.3) that

$$\mathbf{0}=\boldsymbol{\nabla}\times(E\hat{\mathbf{r}})=\frac{1}{r\sin\phi}\frac{\partial E}{\partial\phi}\hat{\boldsymbol{\theta}}-\frac{1}{r}\frac{\partial E}{\partial\theta}\hat{\boldsymbol{\Phi}} \tag{2.3.1}$$

where spherical polar coordinates are used. Thus E is independent of θ and ϕ, and so depends on r and t only.

Now, $\boldsymbol{\nabla}.\hat{\mathbf{r}}=2r^{-1}$, and so equation (2.2) becomes

$$\frac{2}{c}\frac{\partial N}{\partial t}\hat{\mathbf{r}}+2\frac{\partial N}{\partial r}\hat{\mathbf{r}}+\frac{2N}{r}\hat{\mathbf{r}}=\mathbf{0} \tag{2.3.2}$$

It follows immediately that

$$N=\frac{1}{r}f\left(t-\frac{r}{c}\right) \tag{2.3.3}$$

where f is an arbitrary function.

$$\mathbf{D}=-\frac{1}{cr}f\left(t-\frac{r}{c}\right)\hat{\mathbf{r}} \tag{2.3.4}$$

The energy density is

$$\frac{\mu_0}{r^2}\left[f\left(t-\frac{r}{c}\right)\right]^2$$

and the Poynting vector

$$\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\frac{1}{r^2}\left[f\left(t-\frac{r}{c}\right)\right]^2\hat{\mathbf{r}}$$

Thus the energy within a sphere of radius a whose centre is the origin is given by

$$4\pi\mu_0\int_0^a f\left(t-\frac{r}{c}\right)^2 dr \tag{2.3.5}$$

and the rate of flow of energy out across the surface of this sphere is given by

$$4\pi\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\left[f\left(t-\frac{a}{c}\right)\right]^2 \tag{2.3.6}$$

making a equal to zero these will be a source of energy at the origin working at the rate

$$4\pi\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\,[f(t)]^2$$

The energy balance may be discussed in more detail. Consider the quantity

$$\begin{aligned} W_0(a,t) &= 4\pi\mu_0\int_0^a f\left(t-\frac{r}{c}\right)^2 dr \\ &\quad + 4\pi\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\int_{-\infty}^{t}\left\{\left[f\left(t'-\frac{r}{c}\right)\right]^2 - [f(t')]^2\right\}dt' \end{aligned} \tag{2.3.7}$$

The first term on the right-hand side of equation (2.3.7) represents the energy within the sphere $r=a$ at any time, and the second term represents the integral over time of the outflow of energy from this volume.

$W_0(a,t)$ has the following properties

$$W_0(0,t)=0 \tag{2.3.8a}$$

and

$$\frac{\partial W_0}{\partial t}=0 \tag{2.3.8b}$$

It may be regarded, therefore, as an inherent energy associated with the system within the sphere $r=a$. An alternative representation is as follows

$$\begin{aligned} W_0(a,t) &= 4\pi\mu_0\int_0^a\left\{\left[f\left(t-\frac{r}{c}\right)\right]^2 - \int_{-\infty}^{t}\frac{\partial}{\partial t'}\left[f\left(t'-\frac{r}{c}\right)\right]^2 dt'\right\}dr \\ &= 4\pi\mu_0\int_0^a [f(-\infty)]^2\,dr \end{aligned} \tag{2.3.9}$$

which corresponds to the initial energy density, at infinite time past of

$$\mu_0\left[\frac{f(-\infty)}{r}\right]^2$$

The current density is given by

$$\begin{aligned} \mathbf{J} &= \tfrac{1}{2}N(\nabla\cdot\hat{\mathbf{u}})\,\hat{\mathbf{u}} = N(\nabla\cdot\hat{\mathbf{r}})\,\hat{\mathbf{r}} = \frac{N}{r}\hat{\mathbf{r}} \\ &= \frac{1}{r^2}f\left(t-\frac{r}{c}\right)\hat{\mathbf{r}} \end{aligned} \tag{2.3.10}$$

and charge density by

$$\frac{1}{cr^2} f\left(t - \frac{r}{c}\right)$$

the total charge with a sphere of radius a being

$$\frac{1}{4\pi c} \int_0^a f\left(t - \frac{r}{c}\right) dr$$

The charge density creation rate is given by

$$\nabla^2 N - \frac{1}{c^2} \frac{\partial^2 N}{\partial t^2} = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial N}{\partial r}\right) - \frac{1}{c^2} \frac{\partial N}{\partial t^2} = 0 \tag{2.3.11}$$

and there is again apparently no charge creation.

However, the rate of flow of charge, that is the total current out of the sphere of radius a, centre the origin is given by

$$I(a) = 4\pi a^2 [J]_{r=a} = 4\pi f\left(t - \frac{a}{c}\right) \tag{2.3.12}$$

This means that charge flows out of the origin at a rate

$$I(0) = 4\pi f(t)$$

As an example consider an outflow of charge from the origin at the rate $I_0 \cos \omega t$.

The corresponding current density at a distance r is

$$\frac{I_0 \cos \omega[t - (r/c)]}{4\pi r^2} \hat{\mathbf{r}}$$

and the current across the sphere $r = a$ is given by

$$I_0 \cos \omega \left(t - \frac{a}{c}\right)$$

The charge density is

$$\frac{I_0}{4\pi r^2 c} \cos \omega \left(t - \frac{r}{c}\right)$$

and the total charge within a sphere of radius a is given by

$$\frac{I_0}{\omega}\left[\sin \omega t - \sin \omega \left(t - \frac{a}{c}\right)\right]$$

The value of $\mathbf{D}$ is

$$-\frac{I_0}{4\pi rc} \cos \omega \left(t - \frac{r}{c}\right) \hat{\mathbf{r}}$$

and of N

$$\frac{I}{4\pi r}\cos\omega\left(t-\frac{r}{c}\right)$$

The energy density is

$$\frac{\mu_0 I^2}{16\pi^2 r^2}\cos^2\omega\left(t-\frac{r}{c}\right)$$

The total energy with a sphere of radius a is given by

$$W(a)=\int_0^a 4\pi r^2\left[\frac{\mu_0 I^2}{16\pi^2 r^2}\cos^2\omega\left(t-\frac{r}{c}\right)\right]dr=\frac{\mu_0 I^2}{4\pi}\int_0^a \cos^2\omega\left(t-\frac{r}{c}\right)dr$$

$$=\frac{\mu_0 I^2}{8\pi}\left\{a+\frac{c}{2\omega}\left[\sin 2\omega t-\sin 2\omega\left(t-\frac{a}{c}\right)\right]\right\} \tag{2.3.13}$$

The Poynting vector is

$$\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\frac{I^2}{16\pi^2 r^2}\cos^2\omega\left(t-\frac{r}{c}\right)\hat{\mathbf{r}}$$

and the energy flow out across a sphere of radius a is given by

$$P(a)=\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\frac{I^2}{4\pi}\cos^2\omega\left(t-\frac{a}{c}\right) \tag{2.3.14a}$$

implying an energy source creating energy at the rate

$$P(0)=\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}\frac{I^2}{4\pi}\cos^2\omega t \tag{2.3.14b}$$

The expression $W(a)$ may be rewritten in the form

$$W(a,t)=W_0(a)+W_1(a,t)$$

where

$$W_0(a)=\frac{\mu_0 I^2 a}{8\pi} \tag{2.3.15}$$

and

$$W_1(a,t)=\frac{\mu_0 I^2 c}{16\pi\omega}\left[\sin 2\omega t-\sin 2\omega\left(t-\frac{a}{c}\right)\right]$$

$$\frac{dW_1}{dt}=\frac{\mu_0 I^2 c}{8\pi}\left[\cos 2\omega t-\cos 2\omega\left(t-\frac{a}{c}\right)\right] \tag{2.3.16}$$

$$=\frac{\mu_0 I^2 c}{8\pi}\left\{(1+\cos 2\omega t)-\left[1+\cos 2\omega\left(t-\frac{a}{c}\right)\right]\right\}$$

$$=P(0)-P(a)$$

Thus, besides the energy associated with the energy balance equation (2.3.16) there is an unchanging amount of energy given by equation (2.3.15) within the sphere of radius a. It may be remarked that to be consistent with the discussion arising out of equation (2.3.7), it is necessary to replace $\cos\omega t$ by $\exp(-pt^2)\cos\omega t$ ($p > 0$) and take the limit as p tends to zero afterwards, that is, Hardy's theory of generalised integrals is used (Hardy, 1904).

Obviously, a change in sign of c in equation (1.3a) and (1.3b) involves a change in the sign of the flow. It can easily be seen that one of the results of this is that the expression (2.5) for the Poynting vector is the rate of flow of energy in across the surface, instead of the rate of flow out. Correspondingly, the field is associated with a sink of electric charge at the origin, absorbing charge at the rate $4\pi f(t)$. There will be corresponding interpretations in Section 2.2, but these will not be discussed here as they are fairly obvious.

Appendix 1

The energy momentum tensor has not been given by Watson. It may however be calculated easily.

It follows from equation (1.2c) that the force per unit volume is given by $\rho\mathbf{E}+\mathbf{J}\times\mathbf{B}+\mu_0\mathbf{J}N$. Thus the total force on the charge-current system within a volume is given by using equations (1.1) by

$$\begin{aligned}\int\Bigg[&\left(\nabla.\mathbf{D}-\frac{1}{c^2}\frac{\partial N}{\partial t}\right)\mathbf{E}+\left(\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}\right)\times\mathbf{D}\\&+\left(\nabla\times\mathbf{H}-\frac{\partial\mathbf{D}}{\partial t}+\nabla N\right)\times\mathbf{B}+(\nabla.\mathbf{B})\mathbf{H}\\&+\mu_0\left(\nabla\times\mathbf{H}-\frac{\partial\mathbf{D}}{\partial t}+\nabla N\right)N\Bigg]d\tau\end{aligned}\tag{A1.1}$$

$$\begin{aligned}=\int&[(\nabla.\mathbf{D})\mathbf{E}+(\nabla\times\mathbf{E})\times\mathbf{D}+(\nabla.\mathbf{B})\mathbf{H}+(\nabla\times\mathbf{H})\times\mathbf{B}]\,d\tau\\&+\frac{1}{c^2}\frac{\partial}{\partial t}\int(\mathbf{E}\times\mathbf{H}-\mathbf{E}N)\,d\tau\\&+\int\nabla\left(\mu_0\frac{N^2}{2}\right)d\tau+\int\nabla\times(N\mathbf{B})\,d\tau\end{aligned}\tag{A1.2}$$

$$\begin{aligned}=\int&d\mathscr{A}.(\mathbf{T}^{e}+\mathbf{T}^{m})\\&+\int d\mathscr{A}\mu_0\frac{N^2}{2}+\int d\mathscr{A}\times(N\mathbf{B})\\&+\frac{1}{c^2}\frac{\partial}{\partial t}\int[(\mathbf{E}\times\mathbf{H})-\mathbf{E}N]\,d\tau\end{aligned}\tag{A1.3}$$

where $\mathbf{T}^{e}$ and $\mathbf{T}^{m}$ are the electric and magnetic stress tensors (Tralli, 1963).

Possible interpretations of the various terms are as follows: $\mathbf{T}^e$ and $\mathbf{T}^m$ are equivalent to a tension per unit area $\frac{1}{2}\epsilon_0 E^2$ and $\frac{1}{2}(B^2/\mu_0)$ along the parallel to the electric intensity and the magnetic induction respectively and a pressure of the same amount in the directions transverse to them. This is, of course, the usual interpretation. The second term is equivalent to an isotropic pressure $\mu_0(N^2/2)$. The third term is equivalent to a shearing stress across the bounding surface, and the third term to a momentum density

$$\frac{1}{c^2}(\mathbf{E}\times\mathbf{H}-\mathbf{E}N)$$

In the case discussed in this paper $\mathbf{H}$ vanishes and so the shearing stress term vanishes also

$$\mathbf{E}=-\sqrt{\left(\frac{\mu_0}{\epsilon_0}\right)}N\hat{\mathbf{u}}$$

and so $\epsilon_0 E^2=\mu_0 N^2$ and so the momentum density becomes

$$\sqrt{(\mu_0{}^3\,\epsilon_0)}\,N^2\,\hat{\mathbf{u}}$$

and the stress system is equivalent to an isotropic pressure $\frac{1}{2}\mu_0 N^2$, together with a tension per unit area $\frac{1}{2}\mu_0 N^2$ $(=\frac{1}{2}\epsilon_0 E^2)$ parallel to the electric intensity, and a pressure per unit area $\frac{1}{2}\mu_0 N^2$ in directions perpendicular to the electric intensity. The net result of this is a pressure perpendicular to the intensity of amount $\mu_0 N^2$ and no net thrust along the direction of the intensity.

Appendix 2

Suppose that a charge is being created at the point $(0,0,l)$ and destroyed at the point $(0,0,-l)$ at the rate $f(t)$ in both cases. Then if $R_1=|\hat{\mathbf{r}}-l\mathbf{k}|$, $R_2=|\hat{\mathbf{r}}+l\mathbf{k}|$, a possible expression for the associated N field is

$$\frac{1}{4\pi}\left[\frac{F(r_2-ct)}{R_2}-\frac{F(R_1+ct)}{R_1}\right] \tag{A2.1}$$

(As has been pointed out by Watson (1945) the N field can be either outgoing or ingoing.)

Now

$$R_{1,\,2}^2=r^2\mp 2lz+l^2 \tag{A2.2}$$

If l is much larger than r, that is the singularities of charge creation and destruction are at great distances,

$$R_{1,\,2}=l\mp z \tag{A2.3}$$

and the expression (A2.1) for N gives

$$\begin{aligned} N&=\frac{1}{4\pi}\left[\frac{F(l+z-ct)}{l+z}-\frac{F(l-z+ct)}{l-z}\right]\\ &=\frac{1}{4\pi}\left[\frac{F(l+z-ct)-F(l-z+ct)}{l}+0(l^{-2})\right] \end{aligned} \tag{A2.4}$$

Let l tend to infinity in such a way that

$$\frac{1}{4\pi}\frac{F(l+z-ct)-F(l-z+ct)}{l}=f\left(t-\frac{z}{c}\right)$$

is finite.

Equation (A2.4)

$$N=f\left(t-\frac{z}{c}\right)+0(l^{-1})$$

and making l infinite

$$N=f\left(t-\frac{z}{c}\right) \tag{A2.5}$$

Thus, in an N field progressing with the velocity of light in the positive z direction may be thought to be associated with charges being created and destroyed at infinity.

References

Bonnor, W. B. (1969). *International Journal of Theoretical Physics*, Vol. 2, No. 3, p. 373.
Hardy, G. H. (1904). *Quarterly Journal*, **35**, 22.
Tralli, N. (1963). *Classical Electromagnetic Theory*, p. 180. McGraw-Hill, New York.
Watson, W. H. (1945). *Canadian Journal of Physics*, **23A**, 33.